**Property of Time-Invariance (Shift-Invariance) for a System Under Observation**

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When evaluating system properties, we treat a system as a closed box and analyze the relationships between input signals and their corresponding output signals. This process assumes that after inputting a signal, we can return the system to its original state.

A continuous-time system with input signal and output signal is time-invariant (shift-invariant) if whenever the input signal is delayed by seconds, then the output signal will always be delayed by seconds as well for all real values of

A way to visualize the time-invariance property is to show the equivalence between

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That is, does for all possible real constant values of ?

**One-sided infinite observation.** Let’s consider the system under observation for . Time means a time of 0 seconds before occurrence of a Dirac delta occurring at the origin. We can only observe for and for . This means that we can only observe for and for .

**Example.** Consider a delay system that delays the input by *T* seconds, and we can only observe the input signal and the output signal for .

Conceptually, the delay block can be thought as a long wire that conducts electricity from the input to the output. Assuming electrons travel at 2/3 the speed of light, the length of the wire would be (2/3) *c* *T* where *c* is the speed of light (3 x 108 m/s). Such an implementation would be impractical, but nonetheless helpful in analyzing the system.

The first observed output value would be due to the initial conditions in the delay system. In fact, the first *T* seconds of the output would due solely to the initial conditions in the system. For input and output once the initial conditions have been output, . That is, it takes *T* seconds for an input value (voltage) to arrive at the output.

The initial conditions for the delay system consist of the voltage values at different points in the wire at *t* = 0. Let’s denote these voltage values *v*(*t*) for –*T* < *t* ≤ 0. That is, will be first, and will be the last, value among the initial conditions to be output. The spatial location for the voltage *v*(*t*) for –*T* < *t* ≤ 0 is (-2/3) *c* *t* meters from the output location.

Let *x*(*t*) = 0 for 0 ≤ *t* < *T* and 1 for *t* ≥ *T*. For input *x*(*t*), the output is

Let’s keep the same initial conditions, i.e. *v*(*t*) for –*T* < *t* ≤ 0, and the same definition for signal *x*(*t*). Now, we input into the delay system

and the output is

only equals for because the initial conditions did not shift in time even though the input did.

Plots of the signals are given next followed by an analysis of initial conditions:

A picture containing box and whisker chart

Description automatically generatedIf the system were time-invariant, then for all real and . This holds for . For , all unobserved values and initial conditions would have to be equal to a constant value.